

## 2.5 Summary: Conductors

What counts are the *specific* quantities:

- Conductivity  $\sigma$  (or the specific resistivity  $\rho = 1/\sigma$ )
- current density  $j$
- (Electrical) field strength  $\cdot E$

- The basic equation for  $\sigma$  is:  
 $n$  = concentration of carriers  
 $\mu$  = mobility of carriers

- Ohm's law states:  
 It is valid for metals, but not for all materials

$$[\sigma] = (\Omega\text{m})^{-1} = \text{S/m};$$

$$\text{S} = 1/\Omega = \text{"Siemens"}$$

$$[\rho] = \Omega\text{m}$$

$$\sigma = |q| \cdot n \cdot \mu$$

$$j = \sigma \cdot E$$

$\sigma$  (of conductors / metals) obeys (more or less) several rules; all understandable by looking at  $n$  and particularly  $\mu$ .

- Matthiesen rule  
 Reason: Scattering of electrons at defects (including phonons) decreases  $\mu$ .

- " $\rho(T)$  rule":  
 about **0,04 %** increase in resistivity per **K**  
 Reason: Scattering of electrons at phonons decreases  $\mu$

- Nordheim's rule:  
 Reason: Scattering of electrons at **B** atoms decreases  $\mu$

$$\rho = \rho_{\text{Lattice}}(T) + \rho_{\text{defect}}(M)$$

$$\Delta\rho = \alpha_{\rho} \cdot \rho \cdot \Delta T \approx \frac{0,4\%}{^{\circ}\text{C}}$$

$$\rho \approx \rho_A + \text{const.} \cdot [B]$$

Major consequence: You can't beat the conductivity of pure **Ag** by "tricks" like alloying or by using other materials.  
 (Not considering superconductors).

Non-metallic conductors are *extremely* important.

- Transparent conductors (TCO's)  
 ("ITO", typically oxides)
- Ionic conductors (liquid and solid)
- Conductors for high temperature applications; corrosive environments, ..  
 (Graphite, Silicides, Nitrides, ...)
- Organic conductors (and semiconductors)

No flat panels displays = no notebooks etc. without **ITO**!

Batteries, fuel cells, sensors, ...

Example: **MoSi<sub>2</sub>** for heating elements in corrosive environments (dishwasher!).

The future High-Tech key materials?

Numbers to know (order of magnitude accuracy sufficient)

$\rho$ (decent metals) about **2  $\mu\Omega\text{cm}$ .**  
 $\rho$ (technical semiconductors) around **1  $\Omega\text{cm}$ .**  
 $\rho$ (insulators) > **1  $\text{G}\Omega\text{cm}$ .**

No electrical engineering without conductors! Hundreds of specialized metal alloys exist just for "wires" because besides  $\sigma$ , other demands must be met, too:

Money, Chemistry (try **Na!**),  
Mechanical and Thermal properties,  
Compatibility with other materials,  
Compatibility with production  
technologies, ...

Example for unexpected conductors being "best" compromise:

Poly Si, Silicides, **TiN**, **W** in  
integrated circuits

Don't forget Special Applications:

Contacts (switches, plugs, ...);  
Resistors;  
Heating elements; ...

Thermionic emission provides electron beams.  
The electron beam current (density) is given by the *Richardson equation*:

$$j = A \cdot T^2 \cdot \exp - \frac{E_A}{kT}$$

$A_{\text{theo}} = 120 \text{ A} \cdot \text{cm}^{-2} \cdot \text{K}^{-2}$  for free electron gas model  
 $A_{\text{exp}} \approx (20 - 160) \text{ A} \cdot \text{cm}^{-2} \cdot \text{K}^{-2}$

$E_A$  = work function  $\approx (2 - >6) \text{ eV}$

Materials of choice: **W**, **LaB<sub>6</sub>** single crystal

High field effects (tunneling, barrier lowering) allow large currents at low  $T$  from small (nm) size emitter

Needs **UHV!**

There are several thermoelectric effects for metal junctions; always encountered in non-equilibrium.

*Seebeck effect*:

Thermovoltage develops if a metal A-metal B junction is at a temperature different from the "rest", i.e. if there is a temperature gradient

Essential for measuring (high) temperatures with a "thermoelement"  
Future use for efficient conversion of heat to electricity ???

*Peltier effect*:

Electrical current  $I$  through a metal - metal (or metal - semiconductor) junction induces a temperature gradient  $\propto I$ , i.e. one of the junction may "cool down".

Used for electrical cooling of (relatively small) devices. Only big effect if electrical heating ( $\propto I^2$ ) is small.

Electrical current can be conducted by *ions* in

- Liquid electrolytes (like **H<sub>2</sub>SO<sub>4</sub>** in your "lead - acid" car battery); including gels
- Solid electrolytes (= ion-conducting crystals). Mandatory for fuel cells and sensors
- Ion beams. Used in (expensive) machinery for "nanoprocessing".

**Challenge:** Find / design a material with a "good" ion conductivity at room temperature

Basic principle

*Diffusion current*  $j_{\text{diff}}$  driven by concentration gradients  $\text{grad}(c)$  of the charged particles (= ions here) equilibrates with the

*Field current*  $j_{\text{field}}$  caused by the internal field always associated to concentration gradients of charged particles plus the field coming from the outside

$$j_{\text{diff}} = - D \cdot \text{grad}(c)$$

$$j_{\text{field}} = \sigma \cdot E = q \cdot c \cdot \mu \cdot E$$

- Diffusion coefficient  $D$  and mobility  $\mu$  are linked via the Einstein relation;  
concentration  $c(x)$  and potential  $U(x)$  or field  $E(x) = -dU/dx$  by the Poisson equation.

$$\mu = eD/kT$$

$$-\frac{d^2U}{dx^2} = \frac{dE}{dx} = \frac{e \cdot c(x)}{\epsilon \epsilon_0}$$

Immediate results of the equations from above are:

- In equilibrium we find a preserved quantity, i.e. a quantity independent of  $x$  - the electrochemical potential  $V_{ec}$ :
- If you rewrite the equation for  $c(x)$ , it simply asserts that the particles are distributed on the energy scale according to the Boltzmann distribution:
- Electrical field *gradients* and concentration *gradients* at "contacts" are coupled and non-zero on a length scale given by the **Debye length**  $d_{Debye}$
- The Debye length is an extremely important material parameter in "ionics" (akin to the space charge region width in semiconductors); it depends on temperature  $T$  and in particular on the (bulk) concentration  $c_0$  of the (ionic) carriers.
- The Debye length is not an important material parameter in metals since it is so small that it doesn't matter much.

$$V_{ec} = \text{const.} = e \cdot U(x) + kT \cdot \ln c(x)$$

$$c(x) = \exp - \frac{(V(x) - V_{ec})}{kT}$$

$$d_{Debye} = \left( \frac{\epsilon \cdot \epsilon_0 \cdot kT}{e^2 \cdot c_0} \right)^{1/2}$$

The potential difference between two materials (her ionic conductors) in close contact thus...

- ... extends over a length given (approximately) by :
- ... is directly given by the Boltzmann distribution written for the energy:  
(with the  $c_1$  = equilibrium conc. far away from the contact.
- The famous *Nernst equation*, fundamental to ionics, is thus just the Boltzmann distribution in disguise!

$$d_{Debye(1)} + d_{Debye(2)}$$

$$\frac{c_1}{c_2} = \exp - \frac{e \cdot \Delta U}{kT} \quad \text{Boltzmann}$$

$$\Delta U = - \frac{kT}{e} \cdot \ln \frac{c_1}{c_2} \quad \text{Nernst's equation}$$

"Ionic" sensors (most famous the  $ZrO_2$  - based  $O_2$  sensor in your car exhaust system) produce a voltage according to the Nernst equation because the concentration of ions on the exposed side depends somehow on the concentration of the species to be measured.

## Questionaire

All multiple choice questions zu 2. Conductors