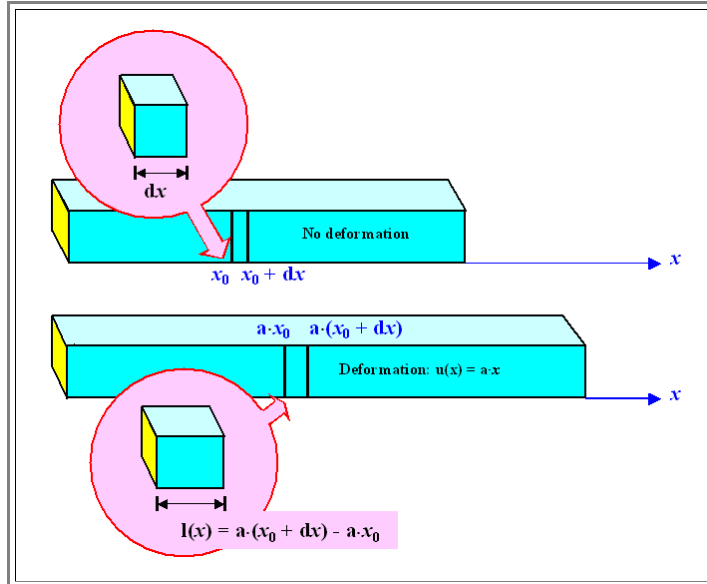


## Displacement and Strain

### Basics

While the relation between the displacement field  $\underline{u}(\underline{r})$  and the local strain tensor  $\epsilon_{ij}$  is rather elementary, it does not hurt to recall the decisive points.

- Let's take the [simple example from the backbone](#) and consider a rod that is uniformly elongated; i.e.  $\underline{u}(\underline{r}) = u_x(\underline{x}) = \mathbf{a} \cdot \underline{x}$ ;  $\mathbf{a}$  is some constant.
- In other words, the vector  $\underline{u}$  *only* has a component in  $\underline{x}$ -direction, which *only* depends on  $\underline{x}$  as variable. The geometry then looks like this:



- At any point in the rod a little cube will be deformed into a **cuboid** - the side in  $\underline{x}$ -direction is somewhat longer than the others.

What kind of strain do we have to put on a cube positioned at  $\underline{x}$ , to produce the cuboid?

- Well, since there is only strain in  $\underline{x}$ -direction, we simply write down the [elementary formula for strain](#)

$$\epsilon_{xx} = \epsilon_x = \frac{l - l_0}{l_0} = \frac{u_x(x + dx) - u_x(x)}{dx} = \frac{du_x}{dx}$$

If we deform in all three directions, we get corresponding expressions for  $\epsilon_{yy}$  and  $\epsilon_{zz}$ .

Since we also might have displacement components in  $\underline{x}$ -direction that depend on  $\underline{y}$  or  $\underline{z}$ , e.g.  $u_x(\underline{x}, \underline{y}, \underline{z}) = \mathbf{a} \cdot \underline{y}$ , we may, in general, also form mixed (partial) derivatives; e.g.  $\partial u_x(\underline{x}, \underline{y}, \underline{z}) / \partial y$ . What do those derivatives signify?

- Shear stresses, of course. A little less easy to see, perhaps, but there can be no doubt about it.
- You may want to try to show that for yourself with the simple displacement field given above and the [equations in the backbone](#) as a guideline for what you are looking for.