

Solution to Exercise 4.1-1 "Lifetime of Positrons"

Illustration

Show that the solution of the differential equations for the positron concentrations n_1 and n_2

$$\frac{dn_1}{dt} = -(\lambda_1 + v \cdot c_V) \cdot n_1$$

$$\frac{dn_2}{dt} = -\lambda_2 \cdot n_2 + v \cdot c_V \cdot n_1$$

The way to obtain the desired equation shall only be sketched.

- First, the coupled differential equation from above need to be solved for the initial condition

$$n_1(t=0) + n_2(t=0) = n_0$$

- n_0 is the number of thermalized positrons in the crystal at the beginning of the experiment: i.e. at $t=0$

This is easy to do since the first differential equation does not contain n_2 .

- The solution must be

$$n_1(t) = A \cdot \exp(-(\lambda_1 + v \cdot c_V)t)$$

Insertion in the second differential equation and using the initial condition yields

$$n(t) = n_0 \cdot \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2 + v \cdot c_V} \exp(-(\lambda_1 + v \cdot c_V)t) + n_0 \cdot \frac{v \cdot c_V}{\lambda_1 - \lambda_2 + v \cdot c_V} \exp(-\lambda_2 t)$$

We have two components decaying with two lifetimes, τ_1 and τ_2 , given by

$$\tau_1 = \frac{1}{\lambda_1 + v \cdot c_V}$$

$$\tau_2 = \frac{1}{\lambda_2}$$

Measurements usually only yield an **average** lifetime $\langle \tau \rangle$

- The **average** lifetime is not simply the average of τ_1 and τ_2 because we need **weighted** averages, i.e.

$$\langle \tau \rangle = \frac{1}{n_0} \int_0^{\infty} t \frac{dn(t)}{dt} dt$$

$$\langle \tau \rangle = \tau_2 \cdot \frac{1 + \tau_2 v \cdot c_V}{1 + \tau_1 v \cdot c_V}$$

 Doing the integral takes a few lines, but it is not too difficult - try it!