Direct Control Methods for Matrix Converter and Induction Machine

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Abstract—Several methods for direct control of matrix converter and induction machine have been developed in recent years. To give an overview and comparison some of them are selected and presented in this paper. Included are the basics of Direct Torque Control, Direct Current Control and Sliding Mode Control of matrix converters.

I. INTRODUCTION

In recent years many research results on control methods for matrix converters have been published. In the field of matrix converter control research work has been done since many years [1]. Commutation and protection issues have been investigated to a high status [2-4]. Early papers concentrated on converter open loop control [1], [5]. Another topic of great interest is the closed loop control of a drive system consisting of matrix converter and induction machine. Slip control and field oriented control are the basic methods applied for closed loop control [4]. Another quite new interesting topic is the application of modern direct control methods to the system with matrix converter and induction machine.

In this paper, the most important direct control methods for matrix converter and induction machine are presented to give an overview and comparison. These methods are Direct Torque Control (DTC), Direct Current Control (DCC) and sliding mode control.

After a short introduction to matrix converter principles and switching constraints in section II an overview of possible switch configurations and their resulting space vectors will be given. In section III the basic ideas of the three control methods mentioned above will be outlined, starting with Direct Torque Control and Sliding Mode Control, as these methods use the same switch configurations of the matrix converter, and ending with Direct Current Control. In section IV similarities and differences of the control methods are mentioned and in section V follows a conclusion.

II. MATRIX CONVERTER PRINCIPLES

The nine bidirectional switches of a matrix converter allow any connection between the three input and output phases as shown in Fig. 1. Some of these connections or switch configurations are forbidden as the voltage sources on the input side must not be short circuited and the inductive load at the output side must not be left open. These constraints lead to 27 allowed switch configurations for the matrix converter which are shown in Table 1 [6]. There and in the following the input phases will be denoted with lowercase indexes a,b,c and output phases with uppercase indexes A,B,C. Table 1 also lists the space vectors of output voltage \( \mathbf{v}_o \) and input current \( \mathbf{i}_a \). These can be assigned to four groups:

Group 1: In this group the output voltage vectors have the same magnitude as the input voltage \( \mathbf{v}_a \), rotate in the same direction and have a displacement angle of 0°, 120° and 240°.

Group 2: In this group the output voltage \( \mathbf{v}_o \) has the same magnitude as \( \mathbf{v}_a \), but rotates in the opposite direction with a displacement angle of 0°, -120° and –240° respectively.

Group 3: In this group the output vectors have a fixed position, but a magnitude varying with a line-to-line input voltage.

Group 4: In this group a zero vector is generated, input and output side of the matrix converter are decoupled.

Depending on the modulation and control method all four groups or only a selection of them are used.

III. DIRECT CONTROL METHODS

A. Direct Torque Control (DTC)

DTC for matrix converters has been designed, analysed and implemented as published in [7], [8] and [9]. This control method uses only the switching combinations from groups 3 and 4 of Table 1. As shown in Fig. 2 a) the output voltage vectors from group 3 form a hexagon with six vectors arranged on one axis of the hexagon. The hexagon is divided into sectors with the output voltage

\[
\mathbf{v}_o = \frac{2}{3} \left( \mathbf{v}_a + \mathbf{v}_b + \mathbf{v}_c \right), \quad \mathbf{a} = e^{(j2\pi/3)} \quad (1)
\]
### Table I  
**Output Voltage and Input Current as Function of Switch Configuration, Input Voltage and Output Current** [6]

<table>
<thead>
<tr>
<th>Switch Configuration</th>
<th>Output Voltage</th>
<th>Input Current</th>
<th>Group</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(v_A) (v_B) (v_C) (v_o)</td>
<td>(i_A) (i_B) (i_C) (i_o)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A) (B) (C)</td>
<td>(A) (B) (C)</td>
<td>(A) (B) (C)</td>
<td>(A) (B) (C)</td>
<td>(A) (B) (C)</td>
</tr>
<tr>
<td>a b c</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b c c</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>a a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>a c b</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b c a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>a a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>a c b</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b c a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>a a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>b a a</td>
<td>2/3 (v_o) 2/3 (v_o) 0</td>
<td>0</td>
<td>(A)</td>
<td>(B)</td>
</tr>
</tbody>
</table>

**Figure 2** Hexagonal arrangements of space vectors. a) Output voltage vectors and sectors; b) input current vectors and sectors; c) output voltage vector direction, stator flux \(\Psi_s\) and changing of stator flux \(\Delta \Psi_s\) within one switching period \(T\) [8]
vectors situated in the middle of each sector. A similar configuration exists for the input current vectors (Fig. 2b), here the current vectors are forming the sector boundaries.

The principle of DTC is to keep stator flux and torque within certain limits by comparing their actual values with the reference values via two hysteresis controllers [10]. Fig. 2c) shows a stator flux vector $\Psi$ within the output voltage hexagon. The small hexagon at the tip of this vector indicates the directions $\Delta \Psi_S$, in which the stator flux vector may be changed within one switching period $T$ by application of one of the voltage vectors which have different directions.

Analog to basic DTC for voltage source converters a direction of output voltage is chosen according to the output voltage sector and the output of the hysteresis comparators for torque and flux $C_T$ and $C_\Psi$, respectively. These directions are given in Table 2, designated with $V_0$ to $V_7$. If the output of the torque hysteresis controller is zero, a zero switch configuration from group 4 is chosen. In all other cases a basic voltage vector direction ($V_1$...$V_6$) is selected.

As there are always two voltage vectors which may be chosen for one given combination of $C_T$ and $C_\Psi$ this gives the possibility to control another quantity. Here the input phase angle is chosen as the two possible voltage vectors are always arranged on the sector boundaries of the corresponding input current sector. The sine of the estimated input phase angle is fed to a third hysteresis comparator. With its output $C_\phi$ and the voltage vector direction from Table 2 the switch configuration which has to be applied to the matrix converter can be taken from Table 3. The numbers in this table denote mode according to Table 1.

### Table II Basic Switching Table for DTC [8, 9]

<table>
<thead>
<tr>
<th>Sector</th>
<th>$C_T=+1$</th>
<th>$C_T=0$</th>
<th>$C_T=-1$</th>
<th>$C_\Psi=+1$</th>
<th>$C_\Psi=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_2$</td>
<td>$V_3$</td>
<td>$V_4$</td>
<td>$V_5$</td>
<td>$V_6$</td>
</tr>
<tr>
<td>2</td>
<td>$V_3$</td>
<td>$V_4$</td>
<td>$V_5$</td>
<td>$V_6$</td>
<td>$V_7$</td>
</tr>
<tr>
<td>3</td>
<td>$V_4$</td>
<td>$V_5$</td>
<td>$V_6$</td>
<td>$V_7$</td>
<td>$V_2$</td>
</tr>
<tr>
<td>4</td>
<td>$V_5$</td>
<td>$V_6$</td>
<td>$V_7$</td>
<td>$V_2$</td>
<td>$V_3$</td>
</tr>
</tbody>
</table>

### Table III Final Switching Table for DTC [8, 9]

<table>
<thead>
<tr>
<th>Sector</th>
<th>$C_\Psi$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$V_5$</th>
<th>$V_6$</th>
<th>$V_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+1</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>+1</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

A block diagram of the complete system is given in Fig. 3. The reference values for torque and flux are compared with the estimated values. The output coefficients of the hysteresis comparators are used together with the sector numbers of stator flux and input voltage vectors to determine the switch configuration according to Tables 2 and 3. The lower part shows the estimators for torque, flux and input phase angle. These require the knowledge of both input and output current and voltage. However, only input voltage and output current are measured, so the remaining values are calculated on the basis of measured values and switch configuration of the matrix converter.

### B. Sliding Mode Control

In [11] and [12] the design of a sliding mode controller for a matrix converter is presented. This controller is able to operate with leading or lagging input power factor and shows a good robustness.

Sliding mode controllers have special interest in systems with variable structure, such as power converters [13, 14]. Their aim is to let the system slide along a predefined sliding surface by changing the system structure.

For designing the sliding mode controller, the source is assumed to be a balanced sinusoidal three phase voltage source with frequency $\omega_0$. The output voltages are assumed to be a similar balanced system with frequency $\omega_0$. The reference values for sliding mode controller design are the output voltages and input currents. For an easier controller design the output voltages are transformed into ‘$\psi$-$\phi$’ coordinates by applying the Concordia transformation. The amplitude of the input current references is calculated from the output currents while the input phase angle is chosen in order to get the desired power factor. The matrix converter real input voltages and reference currents are then transformed into a
reference frame synchronized with the voltage $v_a$ by application of the Park transformation, so the input currents in 'dq' coordinates are received.

To design the sliding mode controller according to \cite{11-14} first the system state space model has to be obtained in phase canonical form. In this form the state variable $x_i$ is represented by a linear combination of all other state variables $x_j$, i.e. \{1,...,n\}. The system is then constrained to have a dynamic $\dot{x}_i$ equal to the linear combination of all $\dot{x}_j$. This results in a sliding surface $S(x,t)$, see (2), which is defined as a linear combination of all $n$ state variables.

$$S(x,t) = \sum_{i=1}^{n} k_i x_i = 0 \quad k_i > 0, i \in \{1,...,n\}$$

(2)

In the special case of the matrix converter the output voltages are directly dependent on the control inputs via the switch combination, so they do not have associated dynamic delays. The sliding surfaces should therefore be dependent on the average values of the ‘ab’ components of the reference voltages. In this case the controller frequency has to be much higher than the desired output frequency in order to guarantee that the average output voltages are equal to their reference values. This results in two sliding surfaces for output voltage control:

$$S(v_{oa},t) = k_{oa} \int_0^T (v_{oa}^* - v_{oa}) dt = 0$$

$$S(v_{ob},t) = k_{ob} \int_0^T (v_{ob}^* - v_{ob}) dt = 0$$

(3)

$k_{oa} > 0$ and $k_{ob} > 0$.

The input currents are like the output voltages discontinuous variables without associated dynamic delays. From this the control laws are similar to those of the output voltages and the sliding surfaces can be obtained similarly. They are expressed as functions of the input currents and their reference values. This results in two sliding surfaces for output voltage control:

$$S(e_{oa},t) = k_{oa} \int_0^T (i_{oa}^* - i_{oa}) dt = 0$$

$$S(e_{ob},t) = k_{ob} \int_0^T (i_{ob}^* - i_{ob}) dt = 0$$

(4)

$k_{oa} > 0$ and $k_{ob} > 0$.

In case the system slides along the defined surfaces it is necessary to guarantee the stability condition

$$S(x,t) \dot{S}(x,t) < 0.$$  \hspace{1cm} (5)

For the designed sliding mode controllers (3, 4), condition (5) can be written as:

$$\frac{k}{T} \int_0^T (x^* - x) dt \cdot (x^* - x) < 0.$$ \hspace{1cm} (6)

This condition is applied to all four sliding mode controllers. It will be verified by following conditions:

a) If $S(e_{oa},t)<0$ then $\dot{S}(e_{oa},t)>0$. This leads to (6): if $\frac{k}{T} \int_0^T (x^* - x) dt < 0$ then $(x^* - x) > 0$, which implies $x < x^*$.

b) If $S(e_{oa},t)>0$ then $\dot{S}(e_{oa},t)<0$. This leads to (6): if $\frac{k}{T} \int_0^T (x^* - x) dt > 0$ then $(x^* - x) < 0$, which implies $x > x^*$.

To guarantee that the system is in sliding mode at each moment a switch configuration has to be chosen, which results in an output voltage vector verifying all four stability constraints. The four sliding surfaces are compared to zero by three level comparators. This results in nine possible error combinations each for output voltages and input currents.

\[e_{oa,d}, e_{ob,d} = \begin{cases} -1 & S(e_{oa},t) < -\epsilon \\ 0 & -\epsilon < S(e_{oa},t) < \epsilon \\ 1 & S(e_{oa},t) > \epsilon \end{cases} \] \hspace{1cm} (7)

From these combinations the control vector is selected, which defines the switch configuration to be applied to the matrix converter. The switch configurations used in this control method are those with fixed angular position and the zero configurations (groups 3 and 4). For practical realisation the comparators are adopted with a hysteresis $\epsilon$ instead of zero in order to reach a bounded switching frequency instead of infinite switching frequency for ideal controller operation.

According to the switching constraints presented in section II it is easy to see: if only output voltage or input current had to be controlled separately there would always be at least one switch configuration which fulfills all error combinations for all reference values. However when both input current and output voltage are considered together sometimes no possible switch configuration is present, especially in case of desired leading or lagging power factors. Another problem arises from the fact that the matrix converter has no intermediate DC link so a chosen switch configuration influences both input current and output voltage. Hence a choice of the switch configuration has to be employed which satisfies both demands to the highest possible degree. This choice has to follow criteria which guarantee controllability of the converter with maximum output voltage and input power factor during minimum output voltage and input current errors.

The choice of the demanded switch configuration starts with output voltage control. From the comparator outputs according to (7) at first a desired sector of output voltage is chosen. In a second decision table the proper switch configuration for this sector is chosen according to the present location of the input voltage vector. For this decision the 18 switch configurations of group 3 with fixed angular position are used. A zero configuration is chosen when all error outputs are zero, the rotating vectors are not considered.

There exists always more than one possible switch configuration for output voltage control, but this is not enough for controlling both components of the input current. However there is chance of choosing a switch configuratio which will satisfy either the $i_d$ or $i_q$ current error. In case that both voltage errors $e_{oa}$ and $e_{ob}$ are equal to zero, full input current control is possible.
For input current control the chosen switch configurations must cause minimum output voltage error in order to minimise output voltage ripple. The basic idea is to maximize the time of \( e_{\text{r,ref}} \) and \( e_{\text{r,obs}} \) equal to zero because this maximizes the input current control range. This leads to large decision tables depending on the four error variables and the sections of input voltage and output current. These tables are omitted here and can be found in [11].

C. Direct Current Control (DCC)

In [15] and [6] a DCC method for matrix converters is presented. It is based on the analysis of the matrix converter’s transfer characteristics. By applying a switching state to the matrix converter a certain voltage space vector \( \mathbf{v}_o \) is generated at the output terminals and vice versa an input current \( i_i \). In a first step the control of the output current \( i_o \) is examined.

In order to minimise the current control error \( \Delta i_o = i_{\text{r,ref}} - i_o \), the output voltage vector \( \mathbf{v}_o \) has to be in the same direction as \( \Delta i_o \) because of the inductive character of the load. In order to achieve this aim the output voltage \( \mathbf{v}_o \) is discretised into six sectors of 60° and the input voltage \( \mathbf{v}_i \) into 12 sections of 30° respectively. For every section the output voltage \( \mathbf{v}_o \) is computed for each switch configuration from Table I and the sector number is put into a first decision table, see Fig. 4. From this table the preferred switching state can be chosen. For example the desired output voltage vector is located in sector 6 because of an output current error in the same sector. If the input voltage vector \( \mathbf{v}_i \) is located in the section between 180° and 210° the switch configurations 2, 5, 13, 14 and 24 produce the desired output voltage. There are always at least three switching states which produce an output voltage vector with the desired direction, so the output current could be easily controlled. However the input current has to be controlled simultaneously.

In order to control the input current a PD-controller is used to determine the sector of the desired input current \( i_i \). This controller is also necessary for active damping of oscillations of the input current which arise from the low cutoff frequency of the input filter and the relatively low switching frequency of the converter. To complete the input current control it has to be taken into account that the output currents of the matrix converter are impressed by the inductive load. The input currents are generated from the output currents via the present switch configuration of the converter.

The input current vector thus can be easily computed from the present output current vector and the switch configuration of the matrix converter. By applying a discretisation as above this results in a second decision table for the input current as a function of switch configuration and output current, which is given in Fig. 5. From this table the desired switch configuration for input current control can be obtained. For example if the desired input current is located in sector 2 and the output current is generated at the output terminals and vice versa an input current \( i_i \). In a first step the control of the output current \( i_o \) is examined.

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vector is in the section between 120° and 150°, switch configurations 6, 11, 14 and 17 are suitable. If at the same time the situation for the output current controller is as in the example above, only mode 14 satisfies both controllers simultaneously.

This results in two direct controllers for both output current $i_o$ and input current $i_e$. As there is not always a switch configuration which satisfies both controller’s demands, a third decision mechanism is applied. The flow diagram of this mechanism is shown in Fig. 6: If both control errors are small, a zero vector (group 4) is selected. If a switch configuration exists which satisfies both controllers simultaneously, this configuration is chosen. In all other cases the weighted errors are compared to each other and the decision table for the controller with the larger error is taken into account. Dependent on the magnitude of the error the switch configuration producing the lowest, medium or largest current amplitude is chosen.

In Fig. 7 a block diagram of the control is given. It shows the PD-controller at the mains side, which determines the desired input current sector and amplitude. The sector of the input voltage vector is determined by a PLL. The output currents are measured and their space vector is determined. From the tables in Fig. 4 and 5 the desired switch configurations are obtained. From these the final switch configuration is determined by Fig. 6, which is then applied to the converter. A more detailed description of the whole control scheme is given in [15].

IV. COMPARISON

The three direct control methods presented in this paper show various differences as well as similarities. So both DTC and sliding mode control use the same switch configurations of groups 3 and 4 (vectors with fixed angular position and zero vectors) while DCC uses the rotating vectors of groups 1 and 2 additionally. Implementations of all three controller types show good steady state an dynamic behaviour. The common main problem is the time consuming process of calculation and decision of the demanded switch configuration. This leads to a high demand for cpu power or restricts implementations to a rather low switching frequency.

The higher number of usable voltage vectors with DTC for matrix converters in comparison to voltage source converters allows to control a third value next to torque and flux. In this case the additional control of the input power factor has been chosen. In addition as presented in [8] the input current quality may be improved by using both possible switch configurations for input power factor control during one sample period, thus including a kind of PWM in DTC.

The presented sliding mode controllers for matrix converters have been tested in their realisation with operation under a wide range of input phase angles. At a step command in input phase angle from −70 to 70 degrees they show a good steady state and dynamic behaviour. The step response shows a fast reaction to the new demands without remarkable overcurrents or voltage distortions.

V. CONCLUSION

Modern direct control methods for matrix converters are presented. The basic ideas of DTC, sliding mode control and DCC for matrix converter and induction machine are outlined as they can be found in literature. All of these methods have been both simulated and realised in lab prototypes by the various authors. Differences and similarities of the selected direct control methods are pointed out.

REFERENCES


